POW #1: Slices of Pie

By Nicole Wiley

You are given a circle and told to cut it into as many pieces as possible for a given number of straight segment cuts. To cut it, you may only use straight lines that intersect the circle twice. You must find the maximum number of slices for 4-10 cuts.

I first went about the problem by drawing a bunch of circles. Then I began drawing lines in the individual circles to make the slices (see attached circle drawings). I quickly realized that to achieve the maximum number of pieces I needed to intersect each line with the other lines. After I found the maximum number of pieces for four and five cuts through drawings, which were eleven and sixteen, I wrote them down in a table, so I could figure out the pattern, which is illustrated in the table. I examined the data in the table. I noticed that the maximum number of pieces from the previous number of cuts plus the next number of cuts resulted in the number of pieces for that cut. Since I now knew the pattern, I calculated the maximum number of pieces for six through ten cuts. I attempted to develop the equation by assigning the number of cuts as the x-value and the number of pieces as the y-value (see page 2 for evidence). Then during math class, Caitlyn showed us how to develop the equations to calculate the maximum number of pieces for a given number of cuts.

The maximum number of pieces for 0-10 cuts is listed in the table. There are three different possible equations for solving this pie slice problem. The first equation is f(n)=f(n-1)+n, where n equals the number of cuts and f(n) equals the maximum number of pieces. The second equation is f(n)=1+$ \sum\_{t=1}^{n}(t)$, where n equals the number of cuts and f(n) equals the maximum number of pieces. The final equation is$ y=.5x^{2}+.5x+1$, where x equals the number of cuts, and y equals the maximum number of pieces. I verified my data using the final equation as I felt that it was the easiest to comprehend. Using these equations, I predict that for 20 cuts the maximum number of pieces is 211, and for 30 it is 466.

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| Number of Cuts | Maximum Number of Pieces | Number Added |
| 0 | 1 |  |
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 3 | 7 | 3 |
| 4 | 11 | 4 |
| 5 | 16 | 5 |
| 6 | 22 | 6 |
| 7 | 29 | 7 |
| 8 | 37 | 8 |
| 9 | 46 | 9 |
| 10 | 56 | 10 |
| 20 | 211 |  |
| 30 | 466 |  |

A similar problem could be trying to achieve the same goal, but using different shapes such as a square. You must find the maximum number of slices for 0-10 cuts. You must also find an equation to figure out the maximum number of pieces.

When I first saw the problem, I thought it sounded very easy, however, it was more difficult than I first perceived. I do consider this POW educationally worthwhile, because through this problem, I now know how to graph on my calculator and how to turn the data from a table into an equation. I would add some guidelines for finding the equation. I felt this problem was slightly too hard, because without Caitlyn’s help, I wouldn’t have found any of the equations. I believe I deserve at least a 23/25. I believe this is a fair score, because I worked very diligently on this problem. I found the correct answers and I have confidence in them.